Experimental Study on SAFETY Estimation of Subgrade Modulus of In-Service Ballasted Tracks in KOREA

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Abstract

The subgrade modulus is an important parameter in the railway safety and the analysis of the behavior of a ballasted track. However, because such analyses often use the design subgrade moduli, their results seldom agree with those of finite element (FE) analyses, which use the theoretically determined subgrade moduli. Moreover, it is difficult to experimentally determine the subgrade modulus and spring stiffness of an in-service track because track components such as the ballast, sleepers, and rails are installed over the subgrade. In this study, the subgrade modulus of an in-service ballasted track was estimated by measuring the dynamic response of the track for railway safety. The subgrade modulus was further predicted from a proposed subgrade modulus map developed from the results of field tests and empirical equations for comparison with the design value. The rail displacement of the ballasted track was also predicted by an FE model that considers the spring stiffness at the rail support point, which includes the subgrade stiffness. It was confirmed that the subgrade modulus of an in-service ballasted track could be reliably predicted on the basis of the dynamic wheel load and rail displacement using the proposed subgrade modulus map.

[Keywords] Railway Safety, Ballasted Track, Subgrade Modulus, Field Test, Finite Element Analysis

1. Introduction

The terms “subgrade modulus”, “ballast modulus”, and “coefficient of ballast” are used interchangeably to describe the same physical parameter, which is the surface pressure load per unit displacement of the loading surface[1]. The subgrade modulus is an important parameter in investigating track safety, deterioration, maintenance, and settlement of ballasted tracks[2][3][4]. It is estimated by a plate load test (PLT) during the preparation of the railway substructure, or subgrade, before the construction of the track. Whereas the characteristics of the subgrade of in-service tracks have not been extensively studied, it is possible to gather relevant information by field measurements. It is difficult to experimentally estimate the subgrade modulus of in-service tracks because the track components such as rails, fastenings, sleepers, and ballast are installed on top of the subgrade. There have been recent attempts to develop a method for measuring the stiffness of the railway subgrade. However, these methods require several field tests and a special test machine and vehicle, which make it very expensive. Moreover, it does not sufficiently consider the behavior of in-service ballasted tracks.

In this study, the subgrade modulus (SM) was determined by an experimental field test on a conventional Korean railway line and compared with that obtained by empirical equations. The SM was thus calculated from the subgrade spring stiffness. Furthermore, a finite element (FE) model of the ballasted track was developed by finite element analysis (FEA).
The track displacement was analyzed and predicted using the FE model. The predictions were compared with the field test results. The theoretically designed SM was first calculated using an empirical equation that had been previously developed from the specifications of the railway subgrade. The measured rail and sleeper displacement, dynamic wheel load, and rail bending stress were substituted into the empirical equation and the result was compared with that estimated from a map of the SM of in-service ballasted tracks developed from the results of field tests.

2. Subgrade Modulus of Ballasted Track

In a ballasted track, the forces generated by the train axle loads are transmitted from the rails, through the sleepers, and to the ballast, foundation, and subgrade. The theoretical model developed by Zimmermann was used to determine the rail displacement[3][4][5]. The model considers the rail as a longitudinal beam that is uniformly and elastically supported at the sleeper support points by the assembly of independent springs that depict the ballast, foundation, and subgrade[3][4][6]. It is also assumed that the deflection of each spring is directly proportional to the generated force[1][4].

It is therefore important to indicate the exact path of a spring when the force and deflection of a ballasted track are discussed[5][6]. DB AG, a German national railway company, classifies springs in terms of the so-called ballast modulus(N/mm³), which is dependent on the rail displacement and surface pressure between the sleeper and the ballast bed[subgrade][6]. On the other hand, a UIC project report[5] classifies springs in terms of the subgrade modulus, C(N/mm³), which is a measure of the vertical surface stiffness of the track support substructure, considering the pressure load and the load-bearing area, which includes the ballast and earthwork layers. On the other hand, a UIC project report[5] classifies springs in terms of the subgrade modulus, C(N/mm³), which is a measure of the vertical surface stiffness of the track support substructure, considering the pressure load and the load-bearing area, which includes the ballast and earthwork layers. Furthermore, the track compendium classifies springs in terms of the coefficient of ballast(N/mm³), which is also a measure of the vertical surface stiffness of the track support substructure determined by the PLT using the prescribed pressure load and surface area of the loading plate on the earthwork layers. The coefficient of ballast indicates the surface pressure load at which the sleeper subsides by 1mm[4]. The modulus can be defined more accurately as

\[ C = \frac{p}{z} \quad (1) \]

Where \( p \) is the surface pressure on a hypothetical load-bearing area(N/mm²) and \( z \) is the vertical rail displacement(mm) induced by the surface pressure. To determine the SM of a particular section of a track, the rail displacement is measured and substituted into Eq.(1), together with the value for \( p \), which is computed from the relevant wheel load and the hypothetical load bearing area of the sleeper[4][6]. The SM describes the stiffness of a support point taking into account the rail bending stiffness and the hypothetical load bearing area[5]. The vertical stiffness of the subgrade \( C_{\text{sub}}(\text{kN/mm}) \) is therefore considered as the spring constant[5]. Generally, the SM or surface stiffness is determined from the PLT, and the subgrade stiffness is obtained by dividing the SM by the hypothetical load bearing area[4][5][6].

\[ C_{\text{sub}} = \frac{C}{A} \quad (2) \]

Where \( C \) is the SM(N/mm³) and \( A \) is the hypothetical load-bearing area(mm²).

Concerning that the total elasticity of the ballasted track is the sum of the elasticity of its various components, the total support point stiffness \( C_{\text{tot}} \), which characterizes the total elasticity below the rail, can be computed by adding the spring constants of the several springs connected in series[3][5][6].

\[ C_{\text{tot}} = \frac{S}{z} \quad (3) \]
Where $S_i$ is the force at the support point of the $l_{hi}$ sleeper(N), and $z$ is the corresponding vertical rail displacement(mm). Because $C_{tot}$ takes into account all the elastic components of the rail support, rigid components such as the concrete sleepers, and concrete structures of the subgrade are not considered in this study.

Only flexible elastic components such as the rail pad, ballast, and subgrade earthwork, which govern the displacement of the ballasted track, are considered. Therefore, the resultant displacement of the ballasted track running through a tunnel or over a bridge is only affected by the displacement of the elastic components between the rail and the concrete surface below. To compare the support point stiffness($C_{tot}$) and the track stiffness($k$), the relationship between the two is determined using Eq.(4)[5][6]. Practical values of the track stiffness are often used to simplify the relationship[5].

$$k = C_{tot} \frac{a}{a} \quad \text{and} \quad C_{tot} = \left( \frac{1}{k_p} + \frac{1}{k_b} + \frac{1}{k_s} \right)^{-1}$$

(4)

Where $a$ is the spacing of the support points(m), $k_p$ is the rail pad stiffness(kN/mm), $k_b$ is the track stiffness(kN/mm), and $k_s$ is the subgrade(stiffness)(kN/mm). The track settlement depends on the sleeper spacing, bending stiffness of the rail, spring stiffness of the rail pad, subgrade type, and subsoil properties. The proposed optimum SM of DB AG is within the very narrow range of 0.05–0.1N/mm³[4][6]. Based on the results of a recent study, the optimum ballasted track stiffness is within the range of 50–100kN/mm. Various empirical equations have been developed for this purpose[4][6]. Because the SM is based on design values, the results of analyses using the theoretically designed SM do not reflect the performance of in-situ ballasted tracks[4][6]. This has made it necessary to develop a method for estimating the SM and stiffness of in-service ballasted tracks to predict their behavior[4][5]. The SM is directly related to the overall track performance, safety, serviceability, and the amount of repair and maintenance required. For this reason, an estimation of the SM is complex and difficult and requires an extensive experimental case study[7][8]. The empirical equations use to determine the SM consider different factors such as rail displacement, rail bending stress, length of the bending wave, and spring stiffness at the rail supporting point, as well as different train and track conditions[4].

<Table 1> lists the various empirical equations used to determine the SM[4].

Table 1. Empirical equations used for determining subgrade modulus[8].

<table>
<thead>
<tr>
<th>N</th>
<th>Method</th>
<th>Empirical equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rail displacement</td>
<td>$C = \frac{Q}{4b\sqrt{\frac{Q}{E_1y}}} = \frac{Q}{2A\sqrt{\frac{Q}{E_1y}}}$</td>
</tr>
<tr>
<td>2</td>
<td>Rail bending stress</td>
<td>$C = \frac{4E-I}{b} \left( \frac{Q}{4\sigma_nW} \right)^4$</td>
</tr>
</tbody>
</table>

Note: $Q$: Static vertical force acting on rail(N), $E$: Modulus of elasticity of rail(N/cm²), $I$: Moment of inertia of rail(cm⁴), $y$: Displacement of rail(cm), $b$: Theoretical rail width(cm), $A$: Half sleeper support surface area(cm²), $a$: sleeper spacing(cm).

This makes the SM dependent on the rail displacement and surface pressure between the sleeper and the ballast bed. The subgrade moduli of old sections range between 0.05 and 0.15N/mm³, whereas those of newly constructed sections range between 0.3 and 0.4N/mm³. The simplest method to determine the SM is to measure the rail displacement and use Eq.(1) in <Table 1>. It is also possible to determine the SM from the bending stresses generated under load at the middle of the rail foot using Eq.(2) in <Table 1>.

3. Field Measurements

Field measurements of the dynamic response of a test track were conducted. A section of an in-service ballasted railway in the Republic of Korea was used for this study. The test section was a straight and continuously welded rail(60kg/m) in the earthwork(operational speed is average of 120km/h). The de-
esign SM\((k_{30})\) of 0.15N/mm\(^3\) was quoted according to the Korean standard(KSF2310) for subgrade materials\([9][10][11]\). The photographs of the test site and vehicle are shown in <Figure 1>.

The dynamic wheel load acting on the track segment was measured by installing a two-axis strain gauge on the rail web between the two test sleepers. The vertical dynamic wheel loads were measured using shear strain gauges coupled to a full Wheatstone bridge circuit\([12]\).

**Figure 1.** Photographs of test track and train.

(a) View of test section. (b) Test train(EMU).

As shown in <Figure 2(b)>, the displacements were measured using displacement transducers(LVDTs) mounted on a jig anchored under the ballast layer of the track.

<Figure 3> shows the dynamic wheel load and rail displacement were affected by the train speed. The sleeper displacements and the rail bending stresses increased slightly with the train speed.

**Figure 2.** Photographs of sensor instrumentation.

(a) Wheel load and rail bending stress. (b) Rail and sleeper displacement.

**Figure 3.** Variation of dynamic response of test track.

(a) Dynamic wheel load. (b) Rail displacement.

4. Prediction of Subgrade Modulus by Qualitative Analysis

In this study, basic concept of qualitative analysis was used to estimate and predict the SM of a real field, which is presented as a SM map. The measured wheel load and rail displacement for the test speed of 120km/h were prepared as reference data(i.e., indicating the range of the discrete space area). The SM was defined as a dependent variable of the qualitative analysis. The following parameter values were adopted based on the results of the field test and design values\([9][10]\).

<Figure 4> shows the variation of the SM with the rail displacement and dynamic wheel load for a vehicle speed of 120km/h. <Figure 4(a)> is a SM diagram that portrays the SM as a function of the dynamic wheel load and rail displacement. The SM of the in-service ballasted track could be predicted from the intersection region of the SM map shown in <Figure 4(b)> and the range of the test results in both the vertical and horizontal directions. In other words, the intersection region of a duplicated zone between the vertical and horizontal directions in <Figure 4(b)> represented the predicted SM of the in-service ballasted track.

**Figure 4.** Subgrade modulus map.

(a) SM vs. dynamic wheel load and rail displacement.
As shown in Figure 4, the discrete space area of the SM decreased with increasing rail displacement and dynamic wheel load. For a rail displacement of less than 0.5mm, the rate of increase in the SM was high. It had a maximum value of 1.2N/mm³ for a dynamic wheel load of 95kN and rail displacement of 0.3mm. As can also be seen from Figure 4, the rail displacement had a greater effect on the SM than the dynamic wheel load. Because the test section of the track was newly constructed (about 2 years old) and the conventional empirical values were taken into account, the SM could be between 0.3 and 0.4N/mm³. However, the results of the predictions based on the measured data ranged between 0.43 and 0.76N/mm³ (the intersection region of a duplicated zone shown in Figure 5(b)). This means that the SM of the in-service ballasted track was higher and more roughly distributed over a wider range than the design value used for the construction.

To ensure an in-service value comparable to the design value, the rail displacement, which is affected by the vertical track stiffness, was kept constant at 0.7mm while the dynamic wheel load was varied. The ballasted track model used for numerical simulation, developed using the FEA package LUSAS. To investigate the train-induced track displacement of the test track, a time-history analysis was performed and the results were compared with those of the field test.

The rails and sleepers comprised frame elements, whereas the rail pad, ballast, and subgrade comprised spring elements. The nodal points between the rail and sleeper elements were connected by spring damper elements with the same properties as those of the rail pad and the ballast. The subgrade conditions under the ballast layer were simulated by a spring element with the same properties as those of the ballast. The subgrade conditions under the ballast layer were simulated by a spring element with the properties listed in Table 2.

### Table 2. Comparison of spring stiffness of subgrade estimated by different methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>SM(N/mm³)</th>
<th>Spring(kN/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design value</td>
<td>0.15</td>
<td>104.0</td>
</tr>
<tr>
<td>Proposed map</td>
<td>0.53–0.58</td>
<td>388.3</td>
</tr>
</tbody>
</table>

Note: Design value obtained by PLT, using the proposed subgrade map with measured data (refer to Figure 5(b)).

The spring stiffness of the rail pad was 400kN/mm and the corresponding properties of the ballast were 200kN/mm, according to the design data. The spring stiffness of the subgrade for the modulus shown in Table 2 was calculated, assuming the hypothetical load bearing area to be 7.628×10⁵mm².

Table 3 compares the SM and vertical rail displacement of the test track obtained by different methods. The measured rail displacements are compared with those of the FEA for the different subgrade moduli in Table 3.

### Table 3. Comparison of FEA and measured results.

<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>SM(N/mm³)</th>
<th>Rail displacement(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A B Test</td>
<td>FEA(1) C FEA(2) D</td>
</tr>
<tr>
<td>82</td>
<td>0.15 0.58</td>
<td>0.461 0.604 0.456</td>
</tr>
<tr>
<td>86</td>
<td>0.15 0.53</td>
<td>0.507 0.634 0.476</td>
</tr>
</tbody>
</table>

Note: A : Design value, B : Obtained using proposed SM map, C : Obtained using design value of SM, D : Obtained using proposed value determined from proposed SM map.

It is interesting to note that the displacements corresponding to the design subgrade moduli by the PLT were greater than those corresponding to the moduli estimated from
the proposed SM map. Consequently, the analytically obtained displacement based on the design SM(FEA(1)) underestimated the behavior of the in-service track.

Moreover, the analytically obtained displacement based on the SM estimated from the proposed SM map(FEA(2)) was less than that of FEA(1). The design SM was approximately 70% less than that estimated from the proposed SM map.

It is therefore considered that the SM directly affects the displacement of an in-service track. Furthermore, the difference between the experimental and FEA(2) displacements was less than that between the experimental and FEA(1) displacements.

The FEA(1) displacement was approximately 1.2 times the experimental displacement, whereas the FEA(2) and experimental displacements were in good agreement with only approximately 5% discrepancy. It is supposed that the FEA results obtained using the SM estimated from the proposed SM map are sufficiently reliable indicators of the behavior of an in-service track.

5. Conclusion

The SM of an in-service ballasted track was assessed by performing field tests using actual vehicles running along service lines. For comparison with the design value, the modulus was predicted using a proposed SM map developed from the results of field tests and empirical equations.

A comparison was performed between results obtained from conventional theory and results of the field test, and the comparison results contributed toward the development of simple estimation methods (not requiring expensive experiments and equipment) for the SM of an in-service ballasted track. The SM of an in-service track can thus be qualitatively predicted by the proposed SM map and a simple field test.

6. References

6.1. Journal articles

6.2. Books

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